

Sistemas de Comunicaciones

Tema 3: Transmisión de Señales



Grado en Ingeniería de Sistemas de Telecomunicación

Departamento de Ingeniería de Comunicaciones
Universidad de Málaga

Curso 2012/2013



OCW UMA

Martos Naya, E; Fernández Plazaola, U.; Cañete Corripio, F.J; Luque Nieto, M.A.
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Tema 3: Transmisión de Señales

3.1 Señales Paso Banda

3.2 Modulaciones Analógicas Lineales

3.3 Modulación Analógica Angular: FM

3.4 Transmisión Digital

Tema 3: Transmisión de Señales

Objetivos

- Representación de señales Paso Banda mediante señales Paso Bajo
- Estudio de Modulaciones Analógicas y evaluación de sus prestaciones
- Conceptos básicos de Modulaciones Digitales

Tema 3: Transmisión de Señales

3.1 Señales Paso Banda

3.2 Modulaciones Analógicas Lineales

3.3 Modulación Analógica Angular: FM

3.4 Transmisión Digital

Tema 3: Transmisión de Señales

3.1 Señales Paso Banda

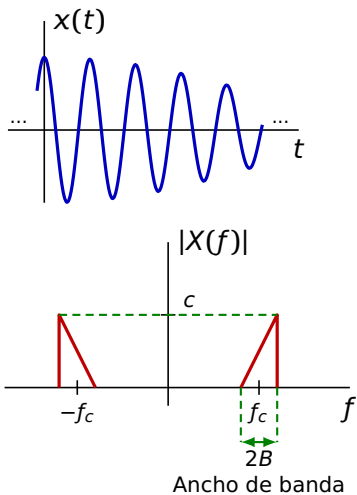
Representación de señales Paso Banda

Modulación de señales

Procesos Paso Banda

3.1 Señales Paso Banda

Señal Paso Banda

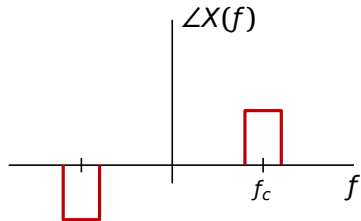


$x(t)$

$x(t) \in \Re$ luego

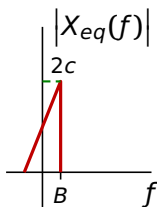
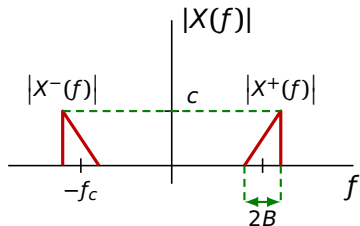
$$X(f) = X^*(-f)$$

$$\begin{cases} |X(f)| = |X(-f)| \\ \angle X(f) = -\angle X(-f) \end{cases}$$



3.1 Señales Paso Banda

Equivalente Paso Bajo



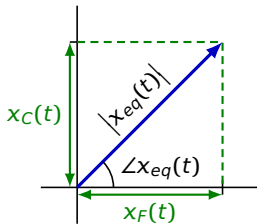
$$X_{eq}(f) = 2X^+(f + f_c)$$

$$X_{eq}(f) \neq X_{eq}^*(-f) \Rightarrow x_{eq}(t) \in \mathbb{C}$$

$$x_{eq}(t) = |x_{eq}(t)| e^{j\angle x_{eq}(t)}$$

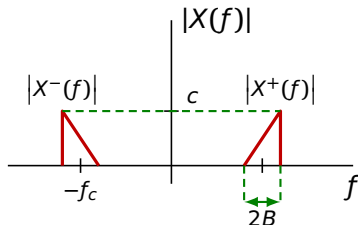
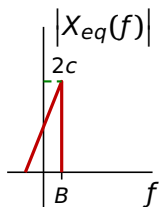
$$x_{eq}(t) = x_F(t) + jx_C(t)$$

$$X_{eq}(f) = X_F(f) + jX_C(f)$$



3.1 Señales Paso Banda

Modulación (frecuencia)



$$X^+(f) = \frac{1}{2}X_{eq}(f - f_c) \quad X^-(f) = (X^+(-f))^* = \frac{1}{2}X_{eq}^*(-f - f_c)$$

$$X(f) = \frac{1}{2} \{ X_{eq}(f - f_c) + X_{eq}^*(-f - f_c) \}$$

$$X(f) = \frac{1}{2} \{ X_F(f - f_c) + jX_C(f - f_c) + X_F(f + f_c) - jX_C(f + f_c) \}$$

3.1 Señales Paso Banda

Modulación (tiempo)

$$X(f) = \frac{1}{2}X_{eq}(f - f_c) + \frac{1}{2}X_{eq}^*(-f - f_c)$$

$$x(t) = \frac{x_{eq}(t)e^{j2\pi f_c t}}{2} + \frac{x_{eq}^*(t)e^{-j2\pi f_c t}}{2}$$

$$x^*(t) \stackrel{\mathcal{F}}{\iff} X^*(-f)$$

$$x(t) = \Re \{ x_{eq}(t) e^{j2\pi f_c t} \}$$

- $x(t) = \Re \{ |x_{eq}(t)| e^{j\angle x_{eq}(t)} e^{j2\pi f_c t} \}$

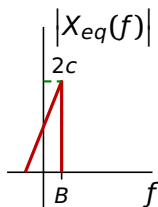
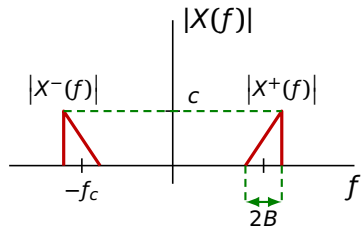
$$x(t) = |x_{eq}(t)| \cos(2\pi f_c t + \angle x_{eq}(t))$$

- $x(t) = \Re \{ [x_F(t) + jx_C(t)] [\cos(2\pi f_c t) + j\text{sen}(2\pi f_c t)] \}$

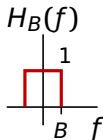
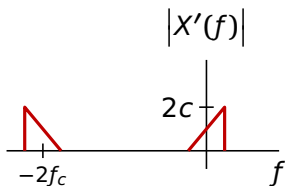
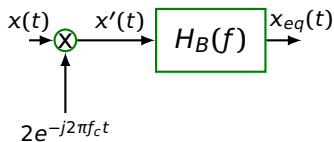
$$x(t) = x_F(t) \cos(2\pi f_c t) - x_C(t) \text{sen}(2\pi f_c t)$$

3.1 Señales Paso Banda

Demodulación (frecuencia)

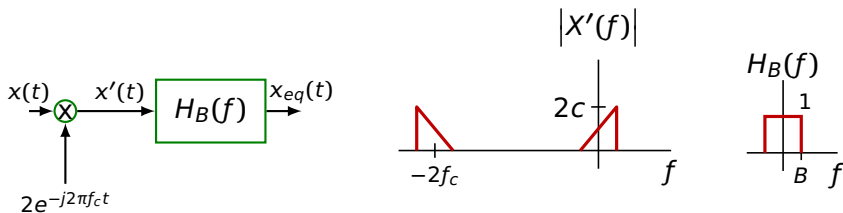


$$X_{eq}(f) = 2X^+(f + f_c)$$



3.1 Señales Paso Banda

Demodulación (tiempo)



$$x_{eq}(t) = [x(t) 2 e^{-j2\pi f_c t}] * h_B(t)$$

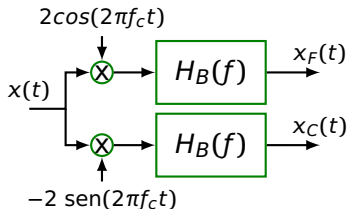
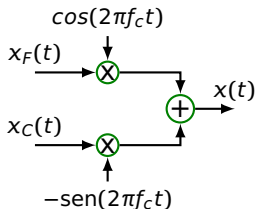
$$x_{eq}(t) = x_F(t) + jx_C(t) = [x(t) 2 (\cos(2\pi f_c t) - j\text{sen}(2\pi f_c t))] * h_B(t)$$

$$x_F(t) = x(t) 2 \cos(2\pi f_c t) * h_B(t)$$

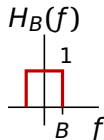
$$x_C(t) = x(t) \cdot -2 \text{sen}(2\pi f_c t) * h_B(t)$$

3.1 Señales Paso Banda

Modulación - Demodulación

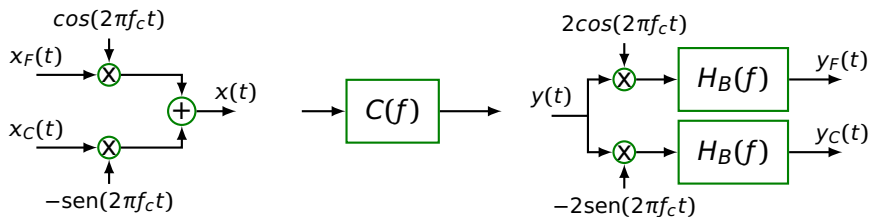


$x(t)$ paso banda de $2B$ de ancho de banda
porta 2 señales paso bajo $x_F(t)$ y $x_C(t)$

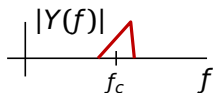
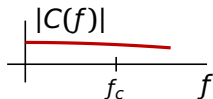
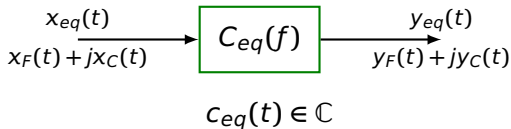


3.1 Señales Paso Banda

Modulación - Canal - Demodulación



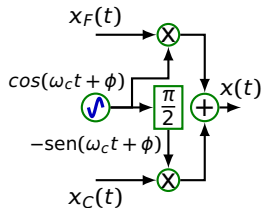
Sistema equivalente paso bajo



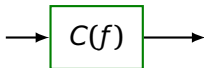
3.1 Señales Paso Banda

Modulación - Canal - Demodulación (más realista)

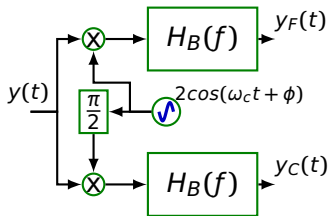
Modulador



Canal



Demodulador



Mismas expresiones pero incluyendo ϕ

$$x(t) = \Re \{ x_{eq}(t) e^{j(2\pi f_c t + \phi)} \} \quad \phi \text{ fase arbitraria}$$

$$x(t) = x_F(t) \cos(2\pi f_c t + \phi) - x_C(t) \text{sen}(2\pi f_c t + \phi)$$

$$y_{eq}(t) = y_F(t) + jy_C(t) = [y(t) 2 e^{-j(2\pi f_c t + \phi)}] * h_B(t)$$

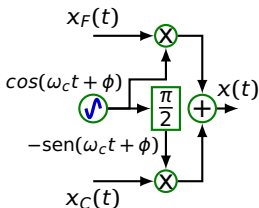
3.1 Señales Paso Banda

Lecturas y ejercicios recomendados

- Ejercicio Relación 3: **3.1**

3.1 Señales Paso Banda

Proceso Paso Banda



$$X(t) = X_F(t) \cos(2\pi f_c t + \Phi) - X_C(t) \sin(2\pi f_c t + \Phi)$$

$X_F(t)$, $X_C(t)$ proc. estacionarios e independientes

$$\overline{X_F(t)} = \overline{X_C(t)} = 0$$

Φ es una vble. aleatoria uniforme en $[-\pi, \pi]$

⇒

$X(f)$ es un proceso estacionario

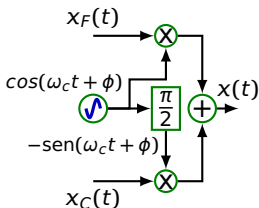
$$S_X(f) = \frac{1}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$

$$S_X(f) = \frac{1}{4} \{S_{X_{eq}}(f - f_c) + S_{X_{eq}}(f + f_c)\}$$

$$P_X = \frac{P_{X_F} + P_{X_C}}{2}$$

3.1 Señales Paso Banda

Proceso Paso Banda (Demostración)



$$X(t) = X_F(t) \cos(2\pi f_c t + \Phi) - X_C(t) \text{sen}(2\pi f_c t + \Phi)$$

$X_F(t)$, $X_C(t)$ estacionarios, indep. y medias nulas

Φ es una vble. aleatoria uniforme en $[-\pi, \pi]$

$\Rightarrow X(f)$ es un proceso estacionario

Demostración

$$R_X(\tau) = E \{X(t)X(t - \tau)\}$$

$$R_X(\tau) = E \{X_F(t)X_F(t - \tau) \cos(2\pi f_c t + \Phi) \cos(2\pi f_c(t - \tau) + \Phi)\} + \\ + E \{X_F(t)X_C(t - \tau) \dots\} + E \{X_C(t)X_F(t - \tau) \dots\} + E \{X_C(t)X_C(t - \tau) \dots\}$$

$$R_X(\tau) = \frac{1}{2} R_{X_F}(\tau) \cos(2\pi f_c \tau) + \frac{1}{2} R_{X_C}(\tau) \cos(2\pi f_c \tau)$$

$$S_X(f) = \frac{1}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$

3.1 Señales Paso Banda

Proceso Paso Banda (Demostración)

$$S_X(f) = \frac{1}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$

$$P_X = \frac{P_{X_F} + P_{X_C}}{2}$$

Por otro lado, $X_{eq}(t) = X_F(t) + jX_C(t)$

$X_F(t)$, $X_C(t)$ estacionarios independientes y medias nulas

\Rightarrow

$$R_{X_{eq}}(\tau) = R_{X_F}(\tau) + R_{X_C}(\tau) \quad \text{y} \quad S_{X_{eq}}(f) = S_{X_F}(f) + S_{X_C}(f)$$

$$P_{X_{eq}} = P_{X_F} + P_{X_C}$$

\Rightarrow

$$S_X(f) = \frac{1}{4} \{S_{X_{eq}}(f - f_c) + S_{X_{eq}}(f + f_c)\}$$

$$P_X = \frac{P_{X_{eq}}}{2}$$

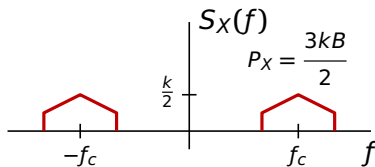
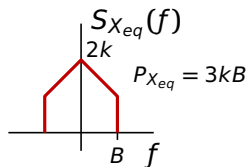
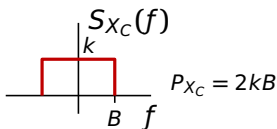
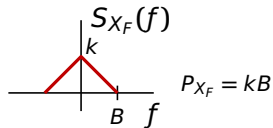
3.1 Señales Paso Banda

Ejemplo proceso paso banda

$$S_{X_{eq}}(f) = S_{X_F}(f) + S_{X_C}(f)$$

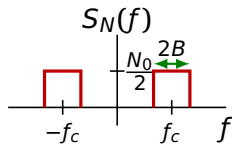
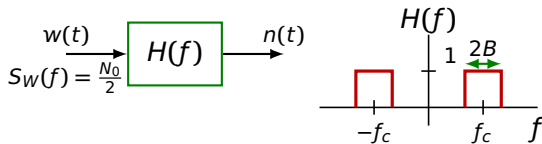
$$S_X(f) = \frac{1}{4} \{S_{X_{eq}}(f - f_c) + S_{X_{eq}}(f + f_c)\}$$

$$S_X(f) = \frac{1}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$



3.1 Señales Paso Banda

Ruido Paso Banda



$W(t)$ Ruido Blanco y gaussiano (AWGN)

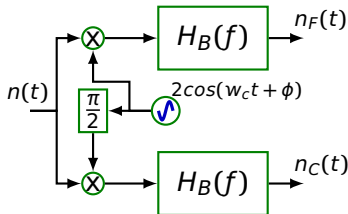
$N(t)$ Ruido Paso Banda gaussiano

Ambos estacionarios y ergódicos

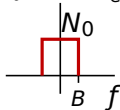
$$S_N(f) = S_W(f) |H(f)|^2$$

\Rightarrow

$$S_{N_F}(f) = S_{N_C}(f) = S_N^+(f + f_c) + S_N^-(f - f_c)$$



$$S_{N_F}(f) = S_{N_C}(f)$$



3.1 Señales Paso Banda

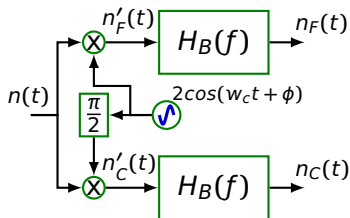
Ruido Paso Banda (Demostración)

$N(t)$ Ruido Paso Banda gaussiano

\Rightarrow

$$S_{N_F}(f) = S_{N_C}(f) = S_N^+(f + f_c) + S_N^-(f - f_c)$$

Demostración:



$$N_F(t) = N(t) 2 \cos(2\pi f_c t + \Phi) * h_B(t) = N'_F(t) * h_B(t)$$

$$N_C(t) = N(t) \cdot -2\text{sen}(2\pi f_c t + \Phi) * h_B(t) = N'_C(t) * h_B(t)$$

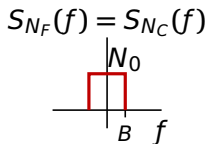
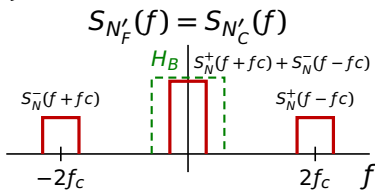
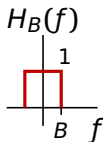
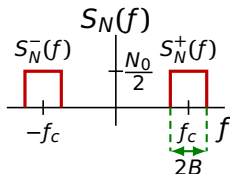
$$R_{N'_F}(\tau) = R_{N'_C}(\tau) = 2R_N(\tau) \cos(2\pi f_c \tau)$$

$$S_{N'_F}(f) = S_{N'_C}(f) = S_N(f - f_c) + S_N(f + f_c)$$

3.1 Señales Paso Banda

Ruido Paso Banda (Demostración)

$$S_{N'_F}(f) = S_{N'_C}(f) = S_N(f - f_c) + S_N(f + f_c)$$



$$S_{N_F}(f) = S_{N'_F}(f) |H_B(f)|^2 = S_N^+(f + f_c) + S_N^-(f - f_c) = S_{N_C}(f)$$

$$P_N = P_{N_F} = P_{N_C} = 2N_0B$$

3.1 Señales Paso Banda

Conclusión

Señal Paso Banda determinista (estudio de señal)

$$x(t) = x_F(t) \cos(2\pi f_c t + \phi) - x_C(t) \sin(2\pi f_c t + \phi)$$

$$X(f) = \frac{1}{2} \{X_F(f - f_c) + jX_C(f - f_c) + X_F(f + f_c) - jX_C(f + f_c)\} \leftarrow (\phi = 0)$$

$$S_X(f) = \frac{1}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$

Señal Paso Banda aleatoria (estudio general de señal o ruido)

$$X(t) = X_F(t) \cos(2\pi f_c t + \Phi) - X_C(t) \sin(2\pi f_c t + \Phi)$$

$$R_X(\tau) = \frac{1}{2} R_{X_F}(\tau) \cos(2\pi f_c \tau) + \frac{1}{2} R_{X_C}(\tau) \cos(2\pi f_c \tau)$$

$$S_X(f) = \frac{1}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$

$$S_{N_F}(f) = S_{N_C}(f) = S_N^+(f + f_c) + S_N^-(f - f_c) \quad (\text{para el ruido})$$

3.1 Señales Paso Banda

Lecturas y ejercicios recomendados

- Capítulo 3. [Haykin3] *Continuous-Wave Modulation* - secciones 3.1 - 3.4 (hasta *Coherent Detection*) (pg. 121-136)
- Capítulo 5. [Haykin3] *Noise in CW Modulation Systems* - secciones 5.2-5.3 (pg. 314-319)

o

- Capítulo 2. [Haykin4] *Continuous-Wave Modulation* - secciones 2.1-2.3 (hasta *Coherent Detection*) (pg. 88-95)
- Capítulo 2. [Haykin4] *Continuous-Wave Modulation* - secciones 2.10-2.11 (pg. 130-132)

3.2 Modulaciones Analógicas Lineales

Introducción

Modulación DBL

Modulación AM

Modulación MQ

El problema de la sincronización

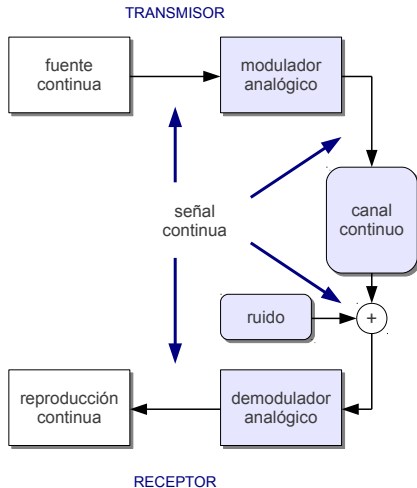
Demodulador incoherente de AM

Otras modulaciones lineales

Receptor Superheterodino

3.2 Modulaciones Analógicas Lineales

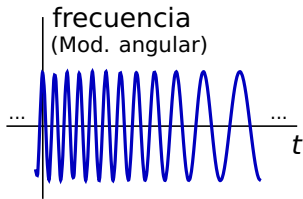
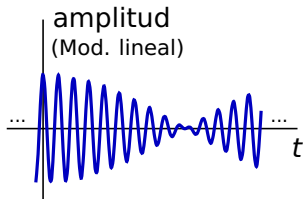
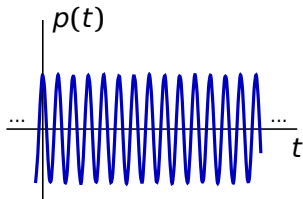
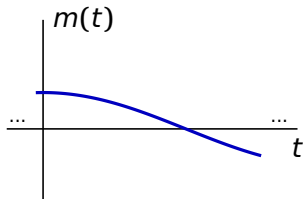
Introducción: Sist. Com. Analógico



3.2 Modulaciones Analógicas Lineales

Introducción

- $m(t)$, señal de información o moduladora
- $m(t)$ varía la amplitud/frecuencia de una portadora $p(t)$



3.2 Modulaciones Analógicas Lineales

Introducción

$$x(t) = Ax_F(t) \cos(2\pi f_c t + \phi) - Ax_C(t) \sin(2\pi f_c t + \phi)$$

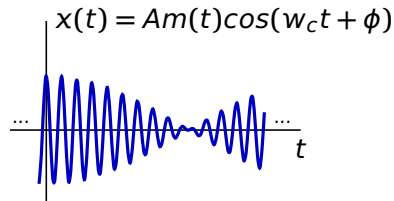
Ejemplos modulaciones lineales

$$\text{DBL: } x_F(t) = m(t) \quad x_C(t) = 0$$

$$\text{AM: } x_F(t) = 1 + K_a m(t) \quad x_C(t) = 0$$

$$\text{MQ: } x_F(t) = m_1(t) \quad x_C(t) = m_2(t)$$

DBL:



3.2 Modulaciones Analógicas Lineales

Introducción

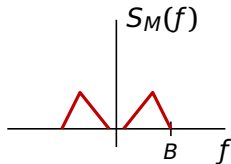
Señal de información

Señal aleatoria $M(t)$ de ancho banda B y media nula ($\overline{M(t)} = 0$)

$$P_M = \int S_M(f) df$$

Otra opción: Señal determinista $m(t)$

$$M(f), S_m(f) \quad P_m = \int S_m(f) df$$



Canal

Ideal $C(f) = c$

Ruido: $W(t)$ proceso estacionario blanco gaussiano y aditivo (AWGN)

$$S_W(f) = \frac{N_0}{2} \left(\frac{W}{\text{Hz}} \right)$$

3.2 Modulaciones Analógicas Lineales

Análisis de prestaciones de modulaciones

SNR en recepción

$$\left(\frac{S}{N}\right) = \frac{\text{Potencia Señal}}{\text{Potencia Ruido}} \text{ (veces)} \quad 10 \log \left(\frac{S}{N}\right) \text{ (dB)}$$

Unidades de potencia: W, dBW, dBm, dB μ

$$\text{dBW} = 10 \log P(W)$$

$$\text{dBm} = 10 \log P(mW)$$

$$\text{dB}\mu = 10 \log P(\mu W)$$

Atenuación del canal (dB): $\alpha = -20 \log c$

$$P_R(\text{dBW}) = P_T(\text{dBW}) - \alpha$$

Eficiencia espectral

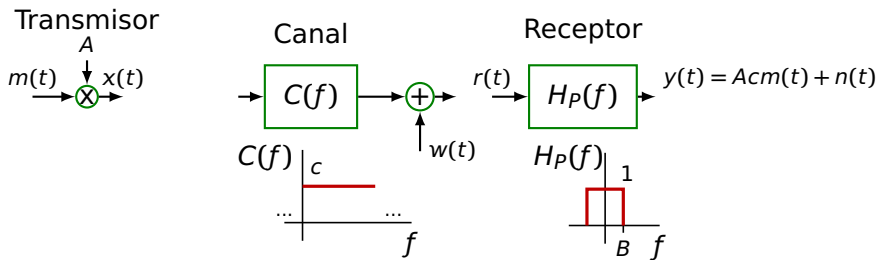
$$\epsilon = \frac{B}{B_T}$$

B : Ancho de banda señal información

B_T : Ancho de banda modulación

3.2 Modulaciones Analógicas Lineales

Prestaciones sin modular



$$P_R = \{\text{solo señal}\} = A^2 c^2 P_M$$

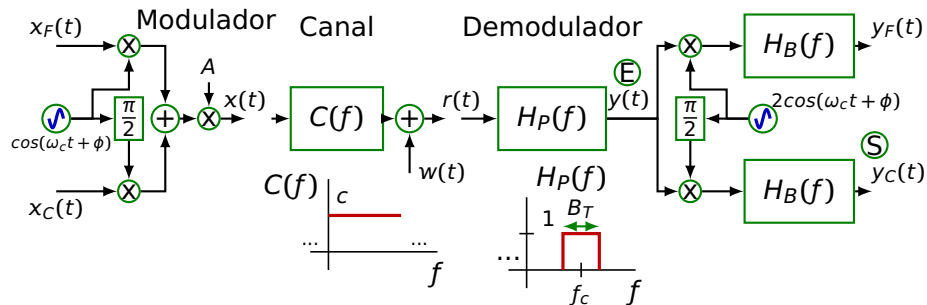
$$P_N = \int_{-B}^B S_W(f) df = \int_{-B}^B \frac{N_0}{2} df = N_0 B$$

$$\left(\frac{S}{N}\right)_{BB} = \frac{P_R}{N_0 B}$$

Prestaciones de referencia

3.2 Modulaciones Analógicas Lineales

Sistema de Comunicaciones Analógico Lineal (I)



$$x(t) = \Re [Ax_{eq}(t)e^{j(\omega_c t + \phi)}] = Ax_F(t)\cos(\omega_c t + \phi) - Ax_C(t)\sin(\omega_c t + \phi)$$

$$r(t) = cx(t) + w(t) \quad y(t) = cx(t) + n(t) \quad A_r = Ac$$

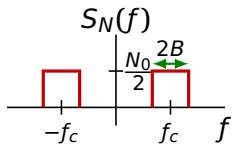
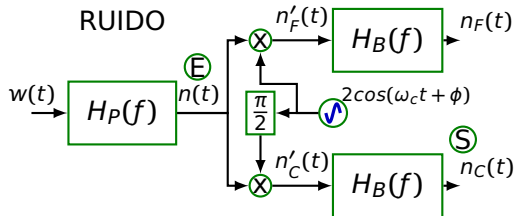
$$y_F(t) = A_r x_F(t) + n_F(t) \quad y_C(t) = A_r x_C(t) + n_C(t)$$

$$S_X(f) = \frac{A^2}{4} \{S_{X_F}(f - f_c) + S_{X_C}(f - f_c) + S_{X_F}(f + f_c) + S_{X_C}(f + f_c)\}$$

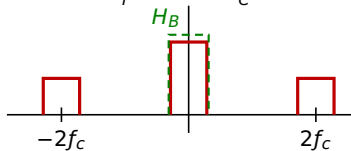
$$P_X = A^2 \frac{P_{X_F} + P_{X_C}}{2} \quad P_Y = \{\text{sólo señal}\} = A_r^2 \frac{P_{X_F} + P_{X_C}}{2}$$

3.2 Modulaciones Analógicas Lineales

Sistema de Comunicaciones Analógico Lineal (II)

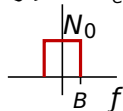
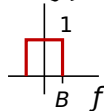


$$S_{N'_F}(f) = S_{N'_C}(f)$$



$$H_B(f)$$

$$S_{N_F}(f) = S_{N_C}(f)$$



$$S_N(f) = S_W(f) |H_P(f)|^2 = \frac{N_0}{2} |H_P(f)|^2$$

$$S_{N'_F}(f) = S_{N'_C}(f) = S_N(f - f_c) + S_N(f + f_c)$$

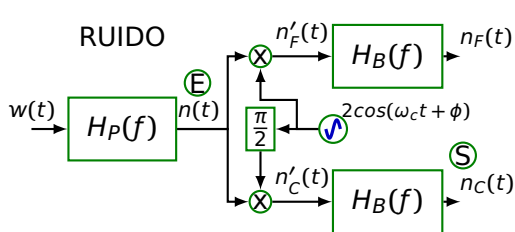
$$S_{N_F}(f) = S_{N_C}(f) = S_N^+(f + f_c) + S_N^-(f - f_c)$$

$$P_{N_F} = P_{N_C} = P_N = 2N_0B$$

$$G_P = \frac{SNR_S}{SNR_E}$$

3.2 Modulaciones Analógicas Lineales

Sistema de Comunicaciones Analógico Lineal (III)



Si los filtros $H_P(f)$ ó $H_B(f)$ no son ideales

$$S_{N(f)} = S_W(f) |H_P(f)|^2$$

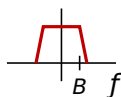
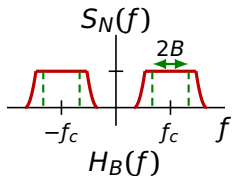
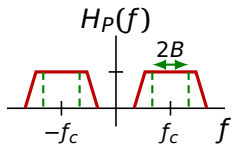
$$S_{N'_F(f)} = S_{N'_C(f)} = S_N(f - f_c) + S_N(f + f_c)$$

$$S_{N_F(f)} = S_{N_C(f)} =$$

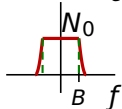
$$= |H_B(f)|^2 \{S_N(f - f_c) + S_N(f + f_c)\}$$

$$P_{N_F} = \int S_N(f) df$$

$$P_{N_F} = P_{N_C} = \int S_{N_F(f)} df$$



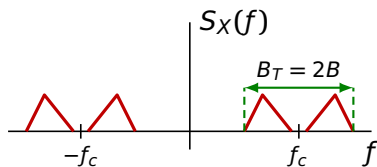
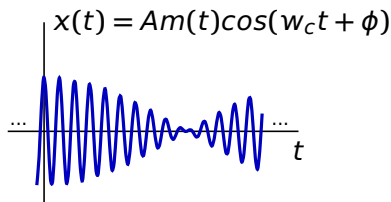
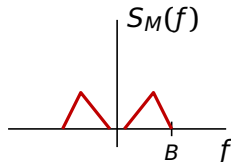
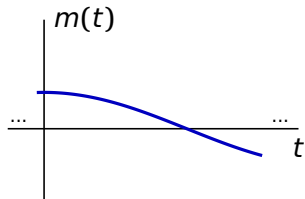
$$S_{N_F(f)} = S_{N_C(f)}$$



3.2 Modulaciones Analógicas Lineales

Modulación DBL

$$x_F(t) = m(t) \quad x_C(t) = 0$$

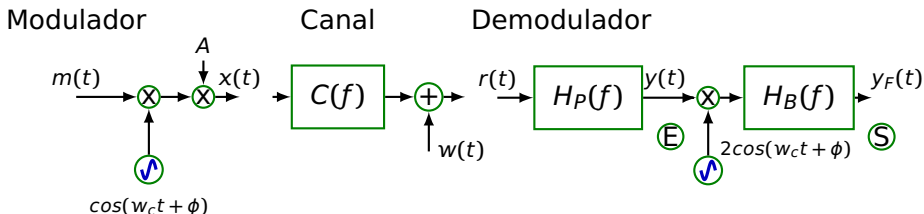


$$S_X(f) = \frac{A^2}{4} \{S_M(f - f_c) + S_M(f + f_c)\}$$

$$P_X = \frac{A^2}{2} P_M$$

3.2 Modulaciones Analógicas Lineales

Modulación DBL: Esquema



$$x_F(t) = m(t) \quad x_C(t) = 0$$

$$x(t) = A m(t) \cos(\omega_c t + \phi) \quad A_r = A c$$

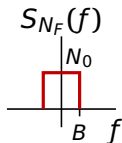
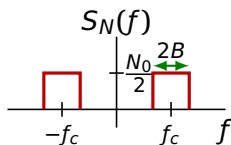
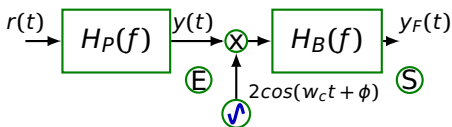
$$r(t) = A_r m(t) \cos(\omega_c t + \phi) + w(t)$$

$$y(t) = A_r m(t) \cos(\omega_c t + \phi) + n(t)$$

$$y_F(t) = A_r m(t) + n_F(t)$$

3.2 Modulaciones Analógicas Lineales

Modulación DBL: SNR



$$r(t) = A_r m(t) \cos(\omega_c t) + w(t)$$

$$y(t) = A_r m(t) \cos(\omega_c t) + n(t)$$

$$y_F(t) = A_r m(t) + n_F(t) \quad P_{N_F} = P_N = 2N_0 B$$

$$P_R = P_Y = \{\text{señal}\} = \frac{A_r^2}{2} P_M \quad P_{Y_F} = \{\text{señal}\} = A_r^2 P_M$$

$$\left(\frac{S}{N}\right)_E = \frac{P_R}{2N_0 B} \quad \left(\frac{S}{N}\right)_S = \frac{A_r^2 P_M}{2N_0 B} = \frac{P_R}{N_0 B} = \left(\frac{S}{N}\right)_{BB} \quad G_P = \frac{\left(\frac{S}{N}\right)_S}{\left(\frac{S}{N}\right)_E} = 2$$

$$B_T = 2B \quad \epsilon = \frac{1}{2}$$

3.2 Modulaciones Analógicas Lineales

Modulación AM

AM \equiv DBL + portadora

$$\begin{aligned}x(t) &= A K_a m(t) \cos(\omega_c t + \phi) + A \cos(\omega_c t + \phi) \\ &= A(1 + K_a m(t)) \cos(\omega_c t + \phi)\end{aligned}$$

$$x_F(t) = 1 + K_a m(t) \quad x_C(t) = 0$$

K_a : Sensibilidad de amplitud

μ : Índice de modulación $\mu = K_a \max |m(t)|$

Asumiremos que $\max |m(t)| = 1 \implies \mu = K_a$

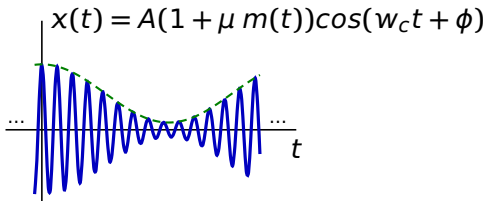
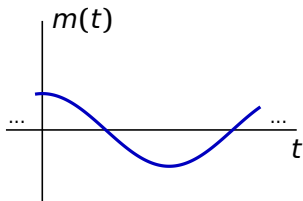
Para que no haya sobremodulación, ha de cumplirse:

$0 < \mu \leq 1 \implies A(1 + \mu m(t)) \geq 0 \implies$ No hay cruces por cero.

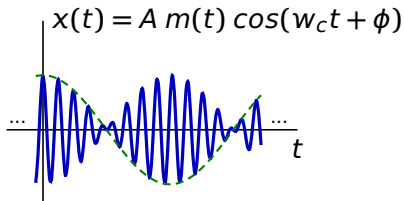
La información $m(t)$ viaja en la envolvente $x(t)$

3.2 Modulaciones Analógicas Lineales

Modulación AM



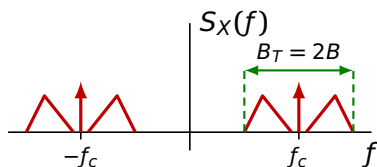
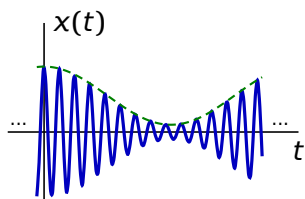
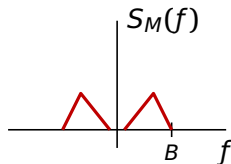
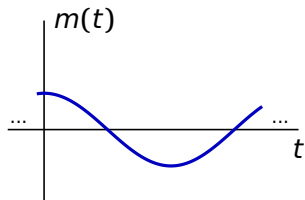
AM



DBL

3.2 Modulaciones Analógicas Lineales

Modulación AM



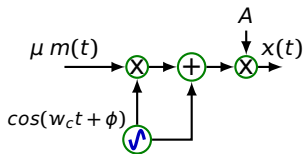
$$S_X(f) = \frac{A^2 \mu^2}{4} \{S_M(f - f_c) + S_M(f + f_c)\} + \frac{A^2}{4} \{\delta(f - f_c) + \delta(f + f_c)\}$$

$$P_X = \frac{A^2}{2} (1 + \mu^2 P_M)$$

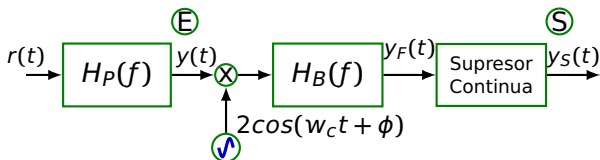
3.2 Modulaciones Analógicas Lineales

Modulación AM: Esquema

Modulador



Demodulador



$$x_F(t) = 1 + \mu m(t) \quad x_C(t) = 0$$

$$x(t) = A(1 + \mu m(t)) \cos(\omega_c t + \phi) \quad A_r = A_c$$

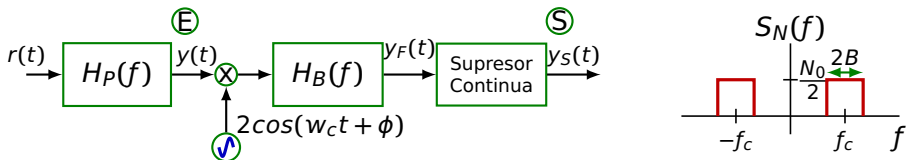
$$y(t) = A_r(1 + \mu m(t)) \cos(\omega_c t + \phi) + n(t)$$

$$y_F(t) = A_r(1 + \mu m(t)) + n_F(t)$$

$$y_S(t) = A_r \mu m(t) + n_F(t)$$

3.2 Modulaciones Analógicas Lineales

Modulación AM: SNR



$$P_R = P_Y = \{\text{señal}\} = \frac{A_r^2}{2}(1 + \mu^2 P_M) \quad P_{N_F} = P_N = 2N_0B$$

$$y_S = A_r \mu m(t) + n_F(t) \quad P_{Y_S} = \{\text{señal}\} = A_r^2 \mu^2 P_M \quad \left(\frac{S}{N}\right)_E = \frac{P_R}{2N_0B}$$

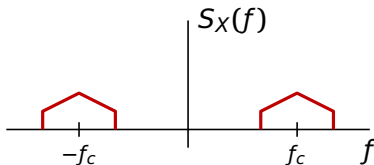
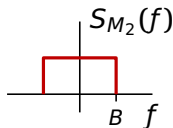
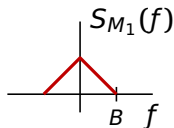
$$\left(\frac{S}{N}\right)_S = \frac{A_r^2 \mu^2 P_M}{2N_0B} = \frac{A_r^2 \mu^2 P_M}{2N_0B} \frac{P_R}{\frac{A_r^2}{2}(1 + \mu^2 P_M)} = \frac{P_R}{N_0B} \left(\frac{\mu^2 P_M}{1 + \mu^2 P_M}\right) < \left(\frac{S}{N}\right)_{BB}$$

$$B_T = 2B \quad \epsilon = \frac{1}{2} \quad G_P = 2 \left(\frac{\mu^2 P_M}{1 + \mu^2 P_M}\right) < 1$$

3.2 Modulaciones Analógicas Lineales

Modulación MQ

$x_F(t) = m_1(t)$ $x_C(t) = m_2(t)$ $M_1(t)$ y $M_2(t)$ independientes

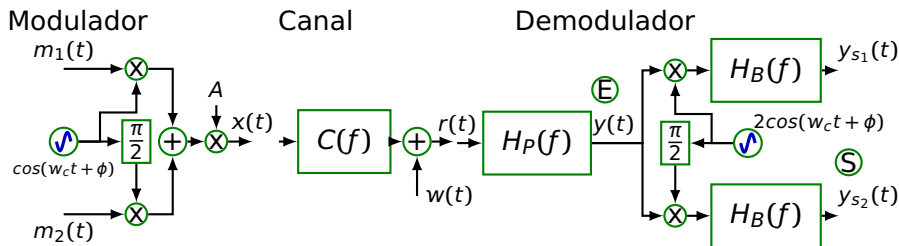


$$S_X(f) = \frac{A^2}{4} \{S_{M_1}(f - f_c) + S_{M_2}(f - f_c) + S_{M_1}(f + f_c) + S_{M_2}(f + f_c)\}$$

$$P_X = A^2 \frac{P_{M_1} + P_{M_2}}{2}$$

3.2 Modulaciones Analógicas Lineales

Modulación MQ: Esquema



$$x_F(t) = m_1(t) \quad x_C(t) = m_2(t)$$

$$x(t) = Am_1(t)\cos(\omega_c t) - Am_2(t)\text{sen}(\omega_c t)$$

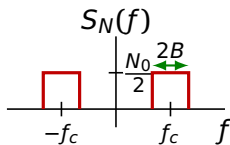
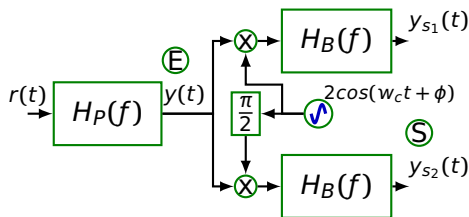
$$y(t) = A_r m_1(t)\cos(\omega_c t) - A_r m_2(t)\text{sen}(\omega_c t) + n(t)$$

$$y_{S1}(t) = A_r m_1(t) + n_F(t)$$

$$y_{S2}(t) = A_r m_2(t) + n_C(t)$$

3.2 Modulaciones Analógicas Lineales

Modulación MQ: SNR



$$P_R = P_Y = \{\text{señal}\} = \frac{A_r^2}{2} (P_{M_1} + P_{M_2}) = A_r^2 P_M$$

$$y_{S_1} = A_r m_1(t) + n_F(t) \quad y_{S_2} = A_r m_2(t) + n_C(t) \quad P_{Y_{S_{1,2}} \text{ señal}} = A_r^2 P_M$$

$$P_{N_F} = P_{N_C} = P_N = 2N_0B \quad \left(\frac{S}{N}\right)_E = \frac{P_R}{2N_0B} \quad B_T = 2B$$

$$\left(\frac{S}{N}\right)_{S_1, S_2} = \frac{A_r^2 P_M}{2N_0B} = \frac{P_R}{2N_0B} = \{P_R/2\} = \left(\frac{S}{N}\right)_{BB} \quad G_P = 1$$

3.2 Modulaciones Analógicas Lineales

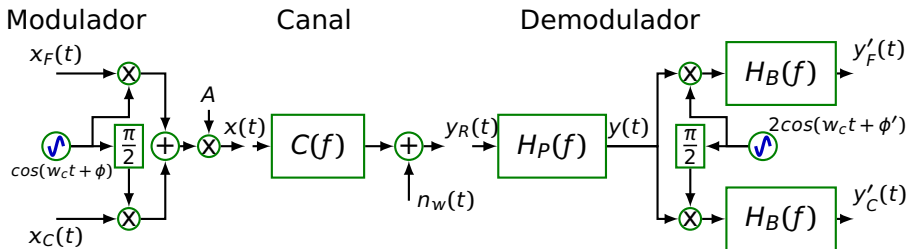
Lecturas y ejercicios recomendados

- Capítulo 3. [Haykin3] *Continuous-Wave Modulation* - sección 3.4 (Desde *Coherent Detection*) (pg. 136 - 140)
- Capítulo 5. [Haykin3] *Noise in CW Modulation Systems* - sección 5.5 (*Noise in AM Receivers*) (pg. 322-325)
- 0
- Capítulo 2. [Haykin4] *Continuous-Wave Modulation* - sección 2.3 (Desde *Coherent Detection* hasta *Television Signals*) (pg. 95 - 101)
- Capítulo 2. [Haykin4] *Continuous-Wave Modulation* - sección 2.12 (*Noise in AM Receivers using Envelope Detection*) (pg. 135-137)

- Ejercicios Relación 3: **3.2, 3.7, 3.10, 3.11**

3.2 Modulaciones Analógicas Lineales

Error de fase en recepción



$$y_{eq}(t) = y_F(t) + jy_C(t) = [y(t) 2 e^{-j(\omega_c t + \phi)}] * h_B(t)$$

$$y'_{eq}(t) = [y(t) 2 e^{-j(\omega_c t + \phi')}] * h_B(t) = [y(t) 2 e^{-j(\omega_c t + \phi)} e^{-j(\phi' - \phi)}] * h_B(t)$$

$$y'_{eq}(t) = y_{eq}(t) e^{-j\theta} \quad \theta = \phi' - \phi$$

$$y'_F(t) = \Re(y'_{eq}(t)) = y_F(t) \cos\theta + y_C(t) \sin\theta \neq y_F(t)$$

$$y'_C(t) = \Im(y'_{eq}(t)) = y_C(t) \cos\theta - y_F(t) \sin\theta \neq y_C(t)$$

3.2 Modulaciones Analógicas Lineales

Error de fase en recepción

DBL/AM

$$y'_F(t) = y_F(t)\cos\theta \quad \left(\frac{S}{N}\right)_{error\ fase} = \left(\frac{S}{N}\right)_i \cos^2\theta$$

MQ

$$y'_{S_1}(t) = y_{S_1}(t)\cos\theta + y_{S_2}(t)\sen\theta$$

$$y'_{S_2}(t) = y_{S_2}(t)\cos\theta - y_{S_1}(t)\sen\theta$$

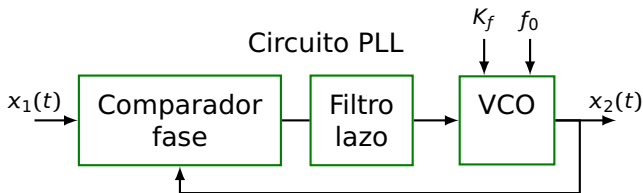
$$\left(\frac{S}{N+I}\right)_{error\ fase} = \frac{S \cos^2\theta}{N + S \sen^2\theta}$$

Error de frecuencia \equiv error de fase variante en el tiempo

3.2 Modulaciones Analógicas Lineales

Tipos de demoduladores

- Demoduladores coherentes.
Requieren circuitos de sincronización (PLL, FLL)
- Demoduladores incoherentes.
No requieren circuitos de sincronización.
Sólo posible para algunas modulaciones (AM)



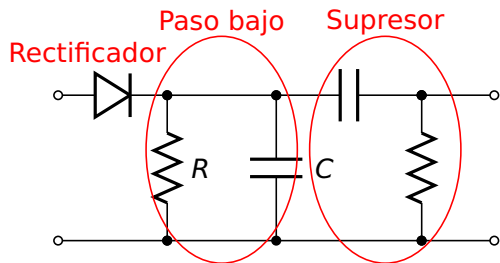
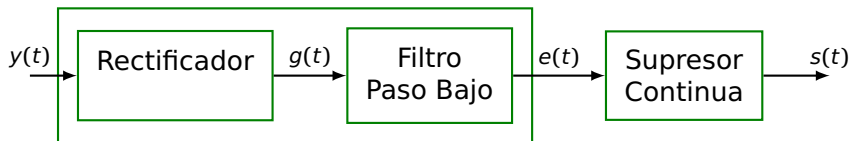
K_f : Sensibilidad de frecuencia (Hz/V) f_0 : frecuencia nominal

Los tonos $x_1(t)$ y $x_2(t)$ se sincronizan en fase

3.2 Modulaciones Analógicas Lineales

Demodulador incoherente de AM

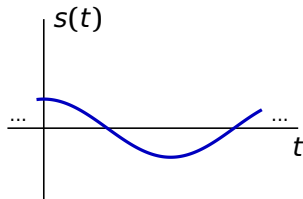
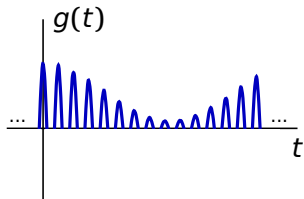
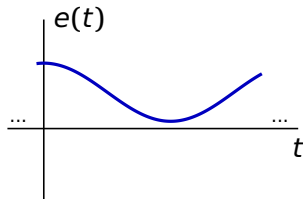
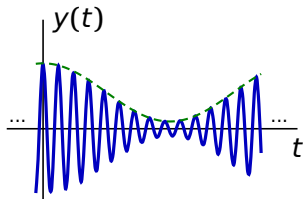
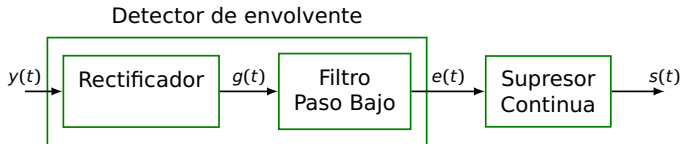
Detector de envolvente



$$B \ll \frac{1}{RC} \ll f_c$$

3.2 Modulaciones Analógicas Lineales

Demodulador incoherente de AM



3.2 Modulaciones Analógicas Lineales

Demodulador incoherente de AM: SNR

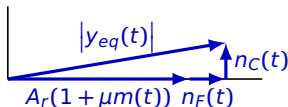
$$y(t) = A_r(1 + \mu m(t))\cos(2\pi fct + \phi) + n(t)$$

$$y(t) = |y_{eq}(t)| \cos(2\pi fct + \angle y_{eq}(t) + \phi) \quad (\text{rep. paso banda})$$

$$y_{eq}(t) = y_F(t) + jy_C(t) = A_r(1 + \mu m(t)) + n_F(t) + jn_C(t)$$

$$e(t) = \{ \text{si } 1 + \mu m(t) > 0 \} = |y_{eq}(t)|$$

$$e(t) = \sqrt{(A_r(1 + \mu m(t)) + n_F(t))^2 + n_C(t)^2}$$



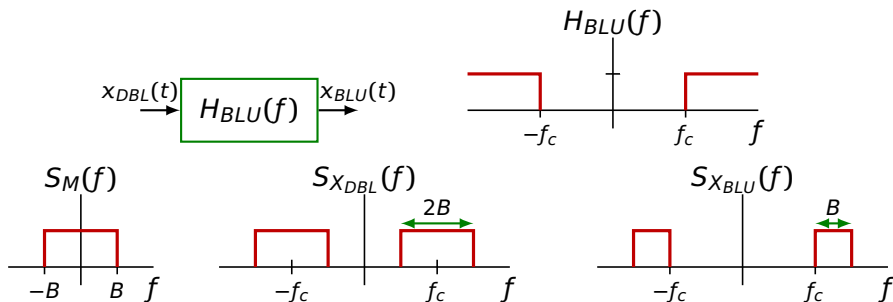
$$e(t) = \{ \text{si } A_r \gg n_F(t), n_C(t) \} \approx A_r(1 + \mu m(t)) + n_F(t)$$

$$\left(\frac{S}{N} \right)_{S_{incoh}} = \left\{ \text{si } \left(\frac{S}{N} \right)_E = \frac{A_r^2(1 + \mu^2 P_M)}{4N_0 B} > \text{Umbral} \right\} = \left(\frac{S}{N} \right)_{S_{coher}}$$

$$\text{Umbral}_{AM} = 13\text{dB}$$

3.2 Modulaciones Analógicas Lineales

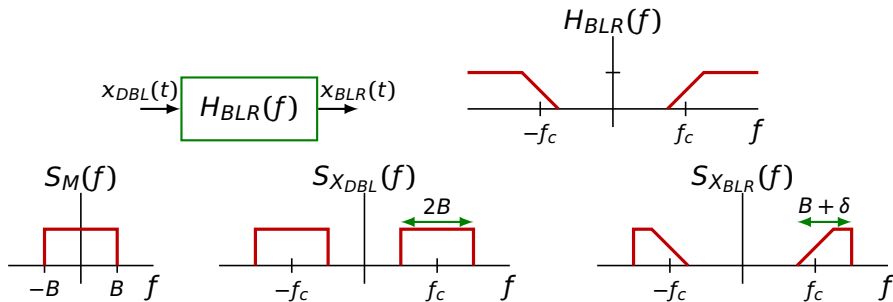
Otras modulaciones: Banda Lateral Única (BLU)



- $B_T = B$ Mitad que DBL
- Filtro abrupto. Realizable si $m(t)$ bajo contenido baja frecuencia
- Sensible a fallos de sincronización
- Usada en telefonía $\left(\frac{S}{N}\right)_{S_{BLU}} = \left(\frac{S}{N}\right)_{BB} = \frac{P_R}{N_0 B}$

3.2 Modulaciones Analógicas Lineales

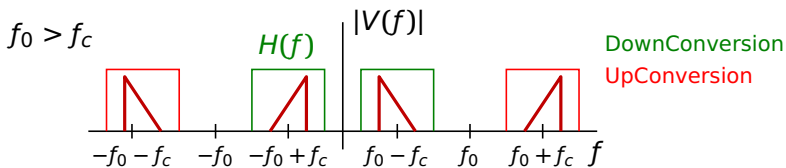
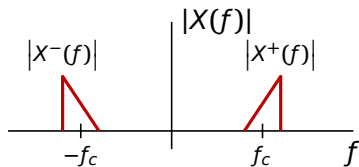
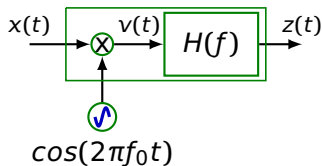
Otras modulaciones: Banda Lateral Residual (BLR)



- $B_T = B + \delta$
- Filtro menos abrupto. Rx más compleja que BLU.
- Sensible a fallos de sincronización
- Usada en Televisión Analógica

3.2 Modulaciones Analógicas Lineales

Mezclador



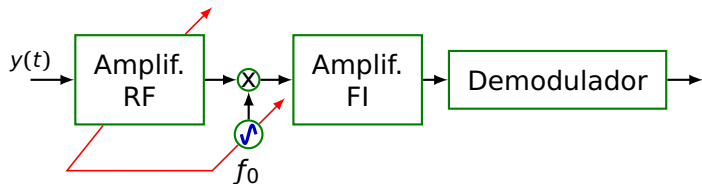
$$x(t) = Ax_F(t) \cos(2\pi f_c t + \phi) - Ax_C(t) \sin(2\pi f_c t + \phi)$$

$$z(t) = Ax_F(t) \cos(2\pi(f_0 - f_c)t + \phi) + Ax_C(t) \sin(2\pi(f_0 - f_c)t + \phi)$$

$$z(t) = Ax_F(t) \cos(2\pi(f_0 + f_c)t + \phi) - Ax_C(t) \sin(2\pi(f_0 + f_c)t + \phi)$$

3.2 Modulaciones Analógicas Lineales

Receptor Superheterodino

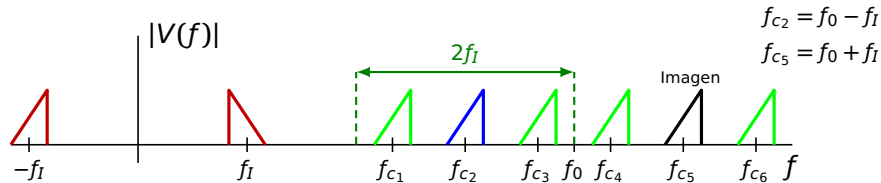


DownConversion.

f_c : Frecuencia canal f_I : Frecuencia intermedia

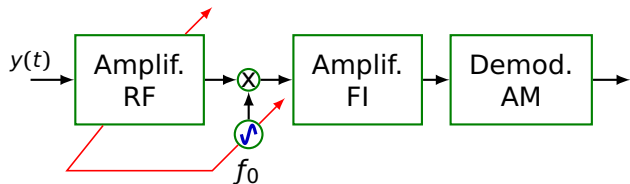
$f_0 = f_c + f_I$: Frecuencia oscilador local (variable)

$B_{RF} < 2f_I$ para evitar el canal imagen ($f_{Imagen} = f_c + 2f_I$)



3.2 Modulaciones Analógicas Lineales

Receptor Superheterodino de AM comercial



AM comercial. Banda: 535-1605 kHz

$f_c = 540 - 1600\text{kHz}$ con 10kHz de separación

$B = 5\text{kHz}$ $f_I = 455\text{kHz}$

$f_0 = f_c + f_I$

3.2 Modulaciones Analógicas Lineales

Lecturas y ejercicios recomendados

- Capítulo 3. [Haykin3] *Continuous-Wave Modulation* - sección 3.10 - 3.11 (*Frequency Modulation*) (pg. 154 - 175)
- Capítulo 5. [Haykin3] *Noise in CW Modulation Systems* - sección 5.6 (*Noise in FM Receivers*) (pg. 326-334)

0

- Capítulo 2. [Haykin4] *Continuous-Wave Modulation* - sección 2.6 - 2.7 (*Frequency Modulation*) (pg. 107 - 121)
- Capítulo 2. [Haykin4] *Continuous-Wave Modulation* - sección 2.13 (*Noise in FM Receivers*) (pg. 142-149)

- Ejercicios Relación 3: **3.12**

Tema 3: Transmisión de Señales

3.3 Modulación Analógica Angular: FM

Ecuaciones de FM

Espectro de un tono modulado

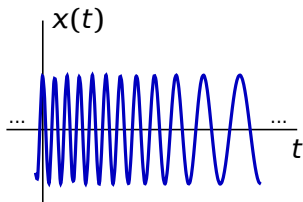
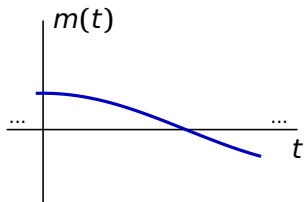
Modulador de FM

Demodulador de FM

FM estereo

3.3 Modulación Analógica Angular: FM

Ecuaciones de FM



$$x(t) = A \cos(2\pi f_c t + \phi(t)) \quad \phi(t) = 2\pi K_f \int_0^t m(\tau) d\tau$$

$$f_i(t) = \left\{ \begin{array}{l} \text{freq.} \\ \text{instan.} \end{array} \right\} = \frac{1}{2\pi} \frac{d(2\pi f_c t + \phi(t))}{dt} = f_c + K_f m(t)$$

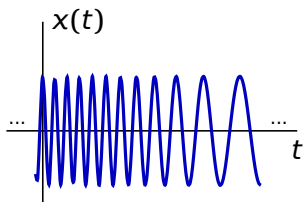
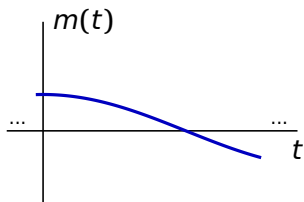
K_f Sensibilidad de frecuencia

$f_\Delta = K_f \max |m(t)|$ Máxima desviación de frecuencia

$\beta = \frac{K_f \max |m(t)|}{B} = \frac{f_\Delta}{B}$ Máxima desviación de fase

3.3 Modulación Analógica Angular: FM

Ecuaciones de FM



$$x(t) = A \cos(2\pi f_c t + \phi(t)) = \Re [A e^{j(2\pi f_c t + \phi(t))}]$$

$$x(t) = \Re [A e^{j\phi(t)} e^{j2\pi f_c t}] = \{\text{recordar}\} = \Re [A x_{eq}(t) e^{j2\pi f_c t}]$$

$$x_{eq}(t) = e^{j\phi(t)} = e^{j2\pi K_f \int_0^t m(\tau) d\tau}$$

Modulación de envolvente constante

3.3 Modulación Analógica Angular: FM

Espectro de un tono modulado

$$m(t) = \cos(2\pi f_0 t)$$

$$\beta = \frac{K_f \max |m(t)|}{B} = \frac{f_\Delta}{B}$$

$$x_{eq}(t) = e^{j\phi(t)}$$

$$\beta_{\text{tono}} = \frac{f_\Delta}{f_0}$$

$$\phi(t) = 2\pi K_f \int_0^t \cos(2\pi f_0 \tau) d\tau = \frac{K_f}{f_0} \text{sen}(2\pi f_0 t) = \beta \text{sen}(2\pi f_0 t)$$

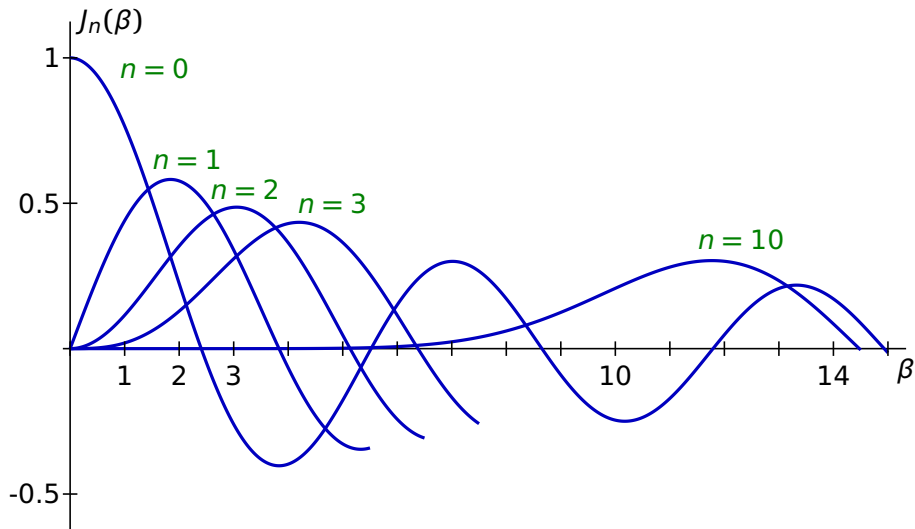
$$x_{eq}(t) = e^{j\beta \text{sen}(2\pi f_0 t)} = \{\text{periódica}\} = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_0 n t}$$

$$C_n = \frac{1}{T_0} \int_{(T_0)} x_{eq}(t) e^{-jn2\pi f_0 t} dt = J_n(\beta)$$

$$x_{eq}(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_0 n t}$$

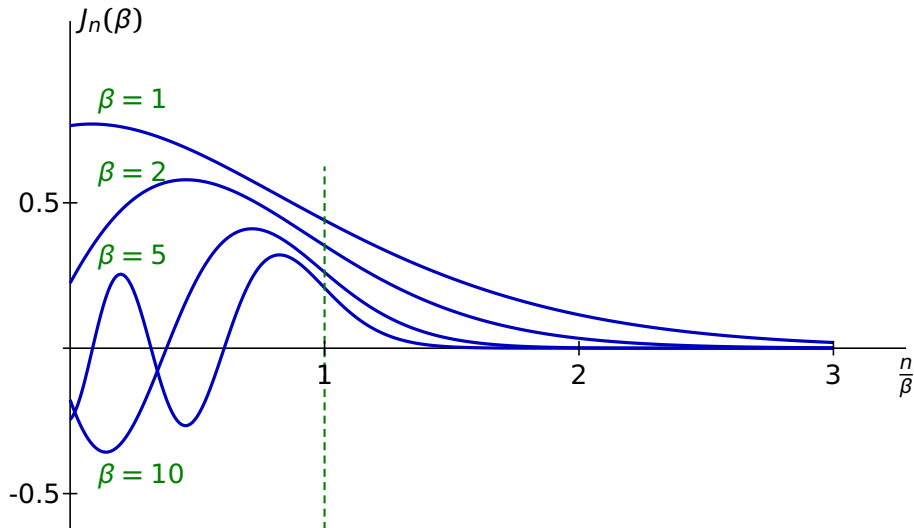
3.3 Modulación Analógica Angular: FM

Espectro tono modulado: Funciones de Bessel



3.3 Modulación Analógica Angular: FM

Espectro tono modulado: Funciones de Bessel



3.3 Modulación Analógica Angular: FM

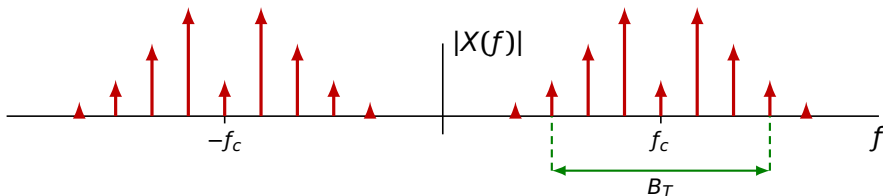
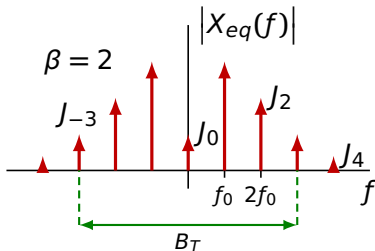
Espectro de un tono modulado

$$J_n(\beta) \simeq 0 \text{ si } |n| > \beta + 1$$

$$B_T = 2(\beta + 1)f_0$$

$$B_T = 2\left(\frac{f_\Delta}{f_0} + 1\right)f_0$$

$$B_T = 2(f_\Delta + f_0)$$



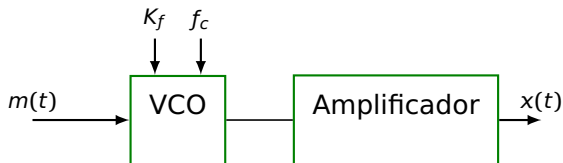
Señal genérica banda B

$$B_T = 2(\beta + 1)B = 2(f_\Delta + B)$$

Regla de Carson

3.3 Modulación Analógica Angular: FM

Modulador de FM



$$x(t) = A \cos \left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau) d\tau \right)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d(2\pi f_c t + 2\pi K_f \int_0^t m(\tau) d\tau)}{dt} = f_c + K_f m(t)$$

VCO con:

- Circuito LC basado en Diodo varactor (C variable)
- Circuitos digitales: *Direct Digital Synthesizer* (DDS)

3.3 Modulación Analógica Angular: FM

Demodulador de FM: solo señal



$$y(t) = A_r \cos \left(2\pi f_c t + 2\pi K_f \int_0^t m(\tau) d\tau \right)$$

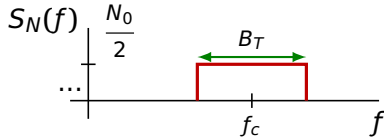
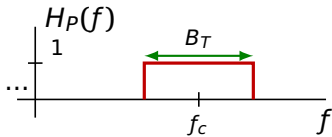
$$y_d(t) = -A_r (f_c + K_f m(t)) \sin \left(2\pi f_c t + 2\pi f_\Delta \int_0^t m(\tau) d\tau \right)$$

$$y_e(t) = A_r (f_c + K_f m(t))$$

$$y_i(t) = A_r K_f m(t)$$

3.3 Modulación Analógica Angular: FM

Demodulador de FM: Análisis de Ruido



$$y(t) = A_r \cos(2\pi f_c t + \phi(t)) + n(t)$$

$$P_R = \frac{A_r^2}{2} \quad P_N = N_0 B_T = N_0 2(\beta + 1)B$$

$$\left(\frac{S}{N}\right)_E = \frac{P_R}{2(\beta + 1)BN_0}$$

3.3 Modulación Analógica Angular: FM

Demodulador de FM: Análisis de Ruido

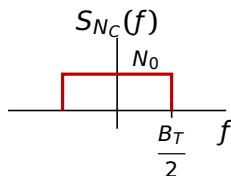
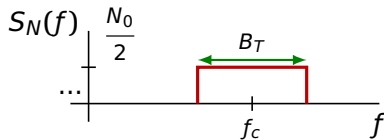


$$y(t) = A_r \cos(2\pi f_c t + \phi(t)) + n(t) = |y_{eq}(t)| \cos(2\pi f_c t + \angle y_{eq}(t))$$

$$y(t) = \left\{ \left(\frac{S}{N} \right)_E > \text{Umbral} \right\} \approx A_r \cos \left(2\pi f_c t + \phi(t) + \frac{n_C(t)}{A_r} \right)$$

$$S_{N_C}(f) = S_N^+(f + f_c) + S_N^-(f - f_c)$$

$$\text{Umbral}_{FM} = 13\text{dB}$$



$$y_i(t) = A_r K_f m(t) + \frac{1}{2\pi} \frac{dn_C(t)}{dt}$$

3.3 Modulación Analógica Angular: FM

Demodulador de FM: Análisis de Ruido



$$y_i(t) = A_r K_f m(t) + n_i(t)$$

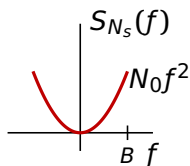
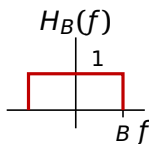
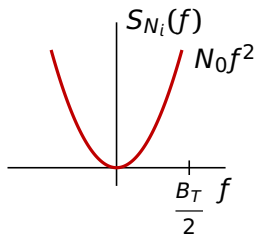
$$n_i(t) = \frac{1}{2\pi} \frac{dn_C(t)}{dt}$$

$$S_{N_i}(f) = S_{N_C}(f) \left| H_{\frac{d}{dt}}(f) \right|^2$$

$$S_{N_i}(f) = N_0 \left| \frac{1}{2\pi} 2\pi f \right|^2 = N_0 f^2 \quad |f| < \frac{B_T}{2}$$

$$y_s(t) = A_r K_f m(t) + n_s(t)$$

$$S_{N_s}(f) = S_{N_i}(f) |H_B(f)|^2$$



3.3 Modulación Analógica Angular: FM

Demodulador de FM: Análisis de Ruido

$$y_s(t) = A_r K_f m(t) + n_s(t)$$

$$P_{Y_s} = A_r^2 K_f^2 P_M = 2P_R K_f^2 P_M$$

$$P_{N_s} = 2 \int_0^B S_{N_s}(f) df = 2 \int_0^B N_0 f^2 df = \frac{2}{3} B^3 N_0$$

$$\left(\frac{S}{N}\right)_S = \frac{P_R}{N_0 B} \frac{3K_f^2 P_M}{B^2} = \frac{P_R}{N_0 B} 3\beta^2 P'_M$$

$$\beta = \frac{K_f \max |m(t)|}{B}$$

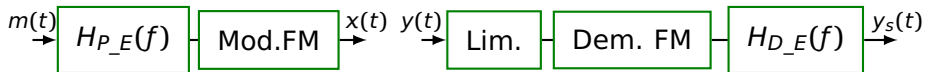
$$P'_M = \frac{P_M}{(\max |m(t)|)^2}$$

$$\frac{\left(\frac{S}{N}\right)_S}{\left(\frac{S}{N}\right)_{BB}} = 3\beta^2 P'_M \quad G_P = \frac{\left(\frac{S}{N}\right)_S}{\left(\frac{S}{N}\right)_E} = 6\beta^2 (\beta + 1) P'_M$$

$$B_T = 2(\beta + 1)B \quad \epsilon = \frac{1}{2(\beta + 1)}$$

3.3 Modulación Analógica Angular: FM

Esquema FM práctico

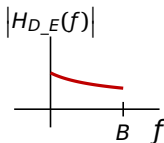
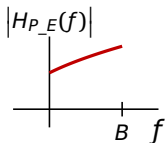


Limitador:

$$A(t) \cos(\omega_c t + \phi(t)) \Rightarrow A \cos(\omega_c t + \phi(t))$$

Pre-énfasis/De-énfasis:

$$H_{D_E}(f) = \frac{1}{H_{P_E}(f)}$$



$$\text{Mejora} = \frac{\left(\frac{S}{N_{D_E}}\right)_S}{\left(\frac{S}{N}\right)_S} = \frac{\frac{2}{3} B^3 N_0}{2N_0 \int_0^B f^2 |H_{D_E}(f)|^2 df} = \frac{B^3}{3 \int_0^B |f H_{D_E}(f)|^2 df}$$

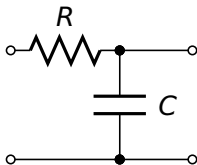
3.3 Modulación Analógica Angular: FM

Pre-énfasis en FM comercial

$$H_{P_E}(f) = 1 + j\frac{f}{f_0} \quad H_{D_E}(f) = \frac{1}{1 + j\frac{f}{f_0}}$$

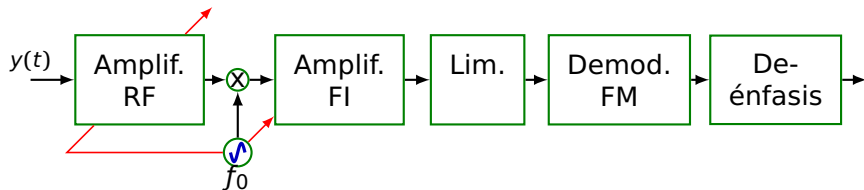
$$f_0 = \frac{1}{2\pi RC} = 2.1\text{kHz}$$

Mejora $\approx 13\text{dB}$



3.3 Modulación Analógica Angular: FM

Receptor Superheterodino de FM comercial



FM comercial. Banda: 88-108 MHz

200kHz de separación entre portadoras

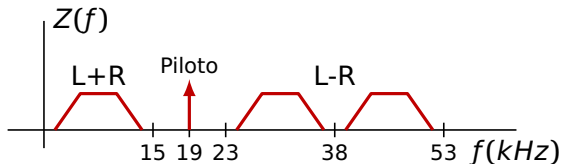
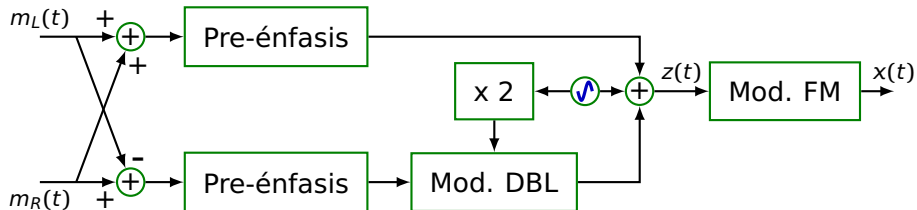
$$f_{\Delta} = 75\text{kHz}$$

$$B = 15\text{kHz} \quad B_T = 2(f_{\Delta} + B) = 180\text{kHz}$$

$$f_I = 10.7\text{MHz} \quad f_0 = f_c + f_I$$

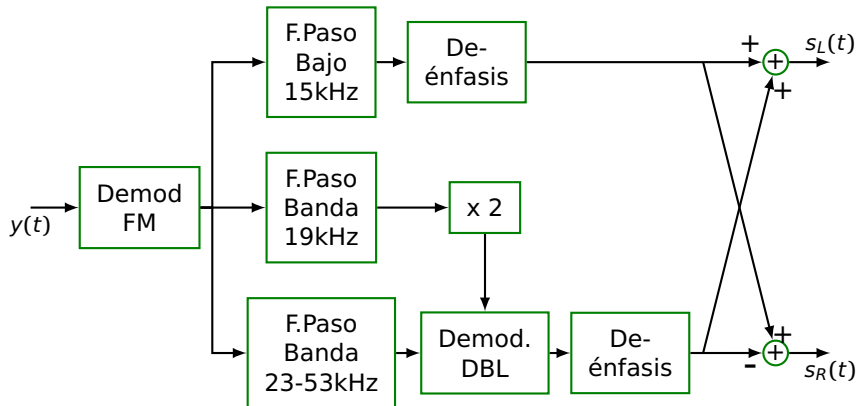
3.3 Modulación Analógica Angular: FM

Transmisor FM estereo



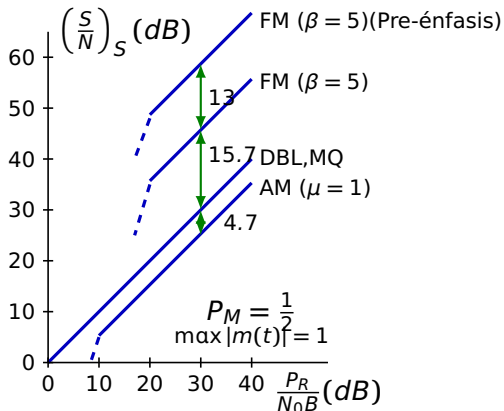
3.3 Modulación Analógica Angular: FM

Receptor FM estereo



3.3 Modulación Analógica Angular: FM

Comparativa de modulaciones



AM comercial :

B = 5 kHz (Voz)

FM comercial (mono):

B = 15 kHz (Audio)

$\beta = 5$

Pre-énfasis (13dB+)